

REAL-WORLD TAKEAWAYS

- Hybrid cars use an electric motor at slower speeds which burns less fuel, causing less air pollution, than a gas powered one.
- Hybrids cars typically cost more up front than their standard versions but require buying less gas to travel the same distance.
- Which car is a better deal depends on how much one drives, how those miles are divided over the city and highway, and how long they plan to keep the car.

MATHEMATICAL OBJECTIVES

- Write and solve equations to solve real-world questions
- Compare proportional relationships represented in different forms including: tables, graphs, and equations

LESSON AT A GLANCE
90 MIN.

		LESSON AT A GLANCE	90 MIN.
LAUNCH		Watch a 2-minute video about how hybrid engines work. Discuss the pros and cons of buying a hybrid vehicle like: they burn less gas, they tend to cost more, etc.	5 min.
PART ONE	1	Calculate how much the average driver would spend on gas in a year for different cars: a hybrid Toyota Camry and the standard version.	25 min.
	2	Determine how long the average driver would have to own the hybrid for it to be a better deal by modeling with tables, graphs, and/or equations.	20 min.
PART TWO	3	For <i>non-average</i> drivers determine how long they'd need to own a hybrid to make it worth the cost. Discuss whether <i>most</i> drivers would be better off buying a hybrid or a standard.	30 min.
	4	Students discuss what factors – environmental impact, gas prices, other – they think influence drivers' decisions about buying hybrids and explain which <i>they</i> would choose.	10 min.

TEACHING NOTE

This lesson has students calculate how much different drivers will spend on gasoline each year. When calculating the solutions in this teacher guide, we use \$3.50/gallon as the price of gasoline. When teaching the lesson, though, you might choose to use the prevailing price in your area. If so, great. In that case, your answers will be different than the ones in this guide.

- L Students watch a video from Toyota explaining how a hybrid engine works. At slow speeds, a battery powers an electric motor which moves the car. At higher speeds – or when a car has to work hard, such as when it’s driving uphill – the traditional gasoline engine takes over.

DISCUSSION QUESTIONS

1. Have you ever heard the expression “hybrid engine”? Can you name any examples of hybrid vehicles?
2. According to the video, how do hybrid engines work?
3. Compared to a gas-only powered car, what might be some advantages of owning a hybrid? Disadvantages?



KEY TAKEAWAYS

- A traditional engine works by burning gasoline. Most cars only use gasoline-powered engines.
- Some car manufacturers have recently begun to offer hybrid vehicles. These include an electric motor for slower speeds and a gasoline-powered engine for faster and harder driving.
- There are pros and cons to owning a hybrid vehicle. They burn less gasoline, but they also tend to cost more.


1 According to AAA, the average American drives around 11,000 miles each year. Imagine a driver is shopping for a new car and is debating between two models of the Toyota Camry: the standard engine (gasoline only) and the hybrid engine (gas + electric).

If the person divides their driving evenly between city and highway, how much will they spend on gas in a year with each model?



ENGINE TYPE		Standard 	Hybrid 
BASE PRICE		\$26,000	\$28,000
FUEL ECONOMY	CITY	28 miles per gallon	52 miles per gallon
	HIGHWAY	39 miles per gallon	52 miles per gallon

PRICE OF GAS



\$3.50

\$ / gallon

METHOD 1: CALCULATE GALLONS USED, THEN TOTAL SPENT ON GAS

	Standard	Hybrid
Annual Gallons, City	$\frac{5,500 \text{ miles}}{28 \text{ miles per gal}} \approx 196 \text{ gallons}$	$\frac{5,500 \text{ miles}}{52 \text{ miles per gal}} \approx 106 \text{ gallons}$
Annual Gallons, Highway	$\frac{5,500 \text{ miles}}{39 \text{ miles per gal}} \approx 141 \text{ gallons}$	$\frac{5,500 \text{ miles}}{52 \text{ miles per gal}} \approx 106 \text{ gallons}$
Annual Gallons, Total	$196 + 141 = 337 \text{ gallons}$	$106 + 106 = 212 \text{ gallons}$
Annual Gas \$	$337 \text{ gallons} \cdot \$3.50/\text{gal.} = \underline{\$1,180}$	$212 \text{ gallons} \cdot \$3.50/\text{gal.} = \underline{\$742}$

METHOD 2: USE DISTRIBUTIVE PROPERTY TO CALCULATE TOTAL SPENT ON GAS

Annual Gas \$, Standard	$\$3.50/\text{gal} \cdot \left(\frac{5,500 \text{ miles}}{28 \text{ mpg}} + \frac{5,500 \text{ miles}}{39 \text{ mpg}} \right) = \$3.50/\text{gal} \cdot (196 \text{ gal} + 141 \text{ gal}) = \$3.50/\text{gal} \cdot 337 \text{ gal} = \$1,180$
Annual Gas \$, Hybrid	$\$3.50/\text{gal} \cdot \left(\frac{5,500 \text{ miles}}{52 \text{ mpg}} + \frac{5,500 \text{ miles}}{52 \text{ mpg}} \right) = \$3.50/\text{gal} \cdot (106 \text{ gal} + 106 \text{ gal}) = \$3.50/\text{gal} \cdot 212 \text{ gal} = \742

TEACHING TIP

While experienced drivers take it for granted, students may not be familiar with the term “miles per gallon.” Before calculating anything, you may want to review what this term means: the number of miles a car is expected to travel on each gallon of gasoline it burns.

- 2 Calculate how many years the average driver would need to own the hybrid before it became the better deal. Based on this, if someone were shopping for a new Camry, which model would you recommend – standard or hybrid – and why?

STRATEGY A: TOTAL COST OF OWNERSHIP

STRATEGY B: TIME TO PAY OFF ADDITIONAL COST

METHOD 1: CREATE TABLE

The hybrid costs more to buy upfront. The standard costs more in gas each year. Want to find number of years until total cost of ownership is the same.

Year	Standard	Hybrid
0	\$26,000	\$28,000
1	\$27,180	\$28,742
2	\$28,360	\$29,484
3	\$29,540	\$30,226
4	\$30,720	\$30,968
5	\$31,900	\$31,710

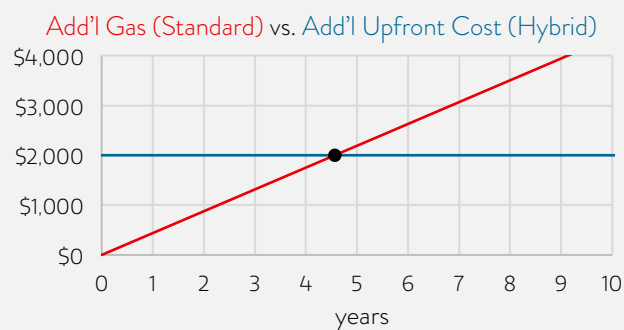
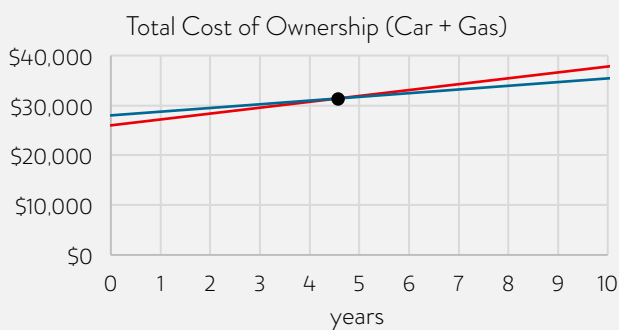
In years 0-4, the standard costs less to own.
The hybrid becomes the less expensive option in year 5.

The hybrid costs \$2,000 more to buy. The standard costs \$438 more in gas each year. Want to know how many years it would take until the extra gas costs “cancels out” the \$2,000.

Year	Add'l Spent on Gas, Standard
0	\$0
1	\$438
2	\$876
3	\$1,314
4	\$1,752
5	\$2,190

The additional amount spent on gas for the standard reaches \$2,000 at some point in year 5.

METHOD 2: SKETCH GRAPH



METHOD 3: WRITE AND SOLVE EQUATION

$$\begin{aligned}
 \text{Standard Cost} &= \text{Hybrid Cost} \\
 26,000 + 1,180x &= 28,000 + 742x \\
 11,780x &= 2,000 + 742x \\
 438x &= 2,000 \\
 x &= 4.6 \text{ years}
 \end{aligned}$$

$$\begin{aligned}
 \text{Standard costs} & \quad \text{Hybrid costs} \\
 \$438 \text{ more in} & \quad \$2,000 \text{ more to} \\
 \text{gas in year} & \quad \text{buy upfront} \\
 438x &= 2,000 \\
 x &= 4.6 \text{ years}
 \end{aligned}$$

The hybrid is more expensive to own until around 4.6 years, at which point the standard becomes more expensive. If someone plans to own a car for more than 4.6 years, it would be cheaper over the long-run to purchase the hybrid engine.

- 3 The average new car lasts around 12 years or 200,000 miles, whichever comes first. Choose one of the drivers below (or create your own). Then make the most convincing argument you can for which type of engine they should buy: standard or hybrid.

DRIVER	ANNUAL MILES, CITY	ANNUAL MILES, HWY.
RETIREE	2,000	2,000
SALESPERSON	4,000	11,000
LYFT/UBER DRIVER	20,000	5,000

PRICE OF GAS



\$3.50 / gallon

STRATEGY A:
CALCULATE YEARS UNTIL HYBRID = STANDARD

STRATEGY B:
CALCULATE LIFETIME COST OF OWNERSHIP

RETIREE

Standard

Total Annual Gallons:

$$(2000 \text{ miles} / 28 \text{ mpg}) + (2000 \text{ miles} / 39 \text{ mpg}) = 122 \text{ gallons}$$

$$\text{Total Annual Gas Cost} = 122 \text{ gal.} \cdot \$3.50/\text{gal.} = \$427$$

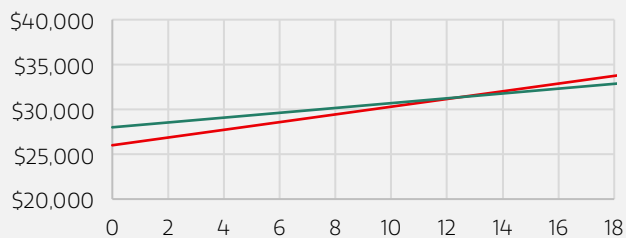
Hybrid

Total Annual Gallons:

$$(2000 \text{ miles} / 52 \text{ mpg}) + (2000 \text{ miles} / 52 \text{ mpg}) = 76 \text{ gallons}$$

$$\text{Total Annual Gas Cost} = 76 \text{ gal.} \cdot \$3.50/\text{gal.} = \$266$$

$$\begin{aligned} \text{Standard Cost} &= \text{Hybrid Cost} \\ 26,000 + 427x &= 28,000 + 266x \\ 161x &= 2,000 \\ x &= 12.4 \text{ years} \end{aligned}$$



It'll take 12.4 years for the hybrid to become the better deal. Since the average car only lasts 12 years, the retiree would save money by choosing the standard engine.

Standard

Total Annual Gallons:

$$(2000 \text{ miles} / 28 \text{ mpg}) + (2000 \text{ miles} / 39 \text{ mpg}) = 122 \text{ gallons}$$

$$\text{Total Annual Gas Cost} = 123 \text{ gal.} \cdot \$3.50/\text{gal.} = \$427$$

Hybrid

Total Annual Gallons:

$$(2000 \text{ miles} / 52 \text{ mpg}) + (2000 \text{ miles} / 52 \text{ mpg}) = 76 \text{ gal.}$$

$$\text{Total Annual Gas Cost} = 76 \text{ gal.} \cdot \$3.50/\text{gal.} = \$266$$

The retiree drives 2,000 + 2,000 = 4,000 miles each year. If they kept the car for 12 years, the car would only have 48,000 miles on it, which is less than 200,000. This means they'd be able to keep the car for the entire 12 years.

$$\begin{aligned} \text{Total Cost of Ownership after 12 Years, Standard:} \\ 26,000 + 427(12) &= \$31,124 \end{aligned}$$

$$\begin{aligned} \text{Total Cost of Ownership after 12 Years, Hybrid:} \\ 28,000 + 270(12) &= \$31,192 \end{aligned}$$

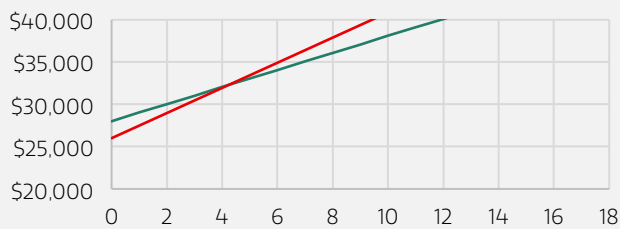
If the retiree owned the car for all 12 years, they'd save a little bit of money (roughly \$70) by choosing the standard engine. Basically, after 12 years, the two cars would cost almost exactly the same amount for the retiree to own.

SALESPERSON

Standard, Total Annual Gallons = 425 gallons
Total Annual Gas Cost = \$1,488

Hybrid, Total Annual Gallons = 289 gallons
Total Annual Gas Cost = \$1,012

$$\begin{aligned}\text{Standard Cost} &= \text{Hybrid Cost} \\ 26,000 + 1,488x &= 28,000 + 1,012x \\ 476x &= 2,000 \\ x &= 4.2 \text{ years}\end{aligned}$$



It'll take 4.2 years for the hybrid to become the better deal. Since the average car lasts 12 years, the salesperson would save a lot of money by choosing the hybrid engine.

Standard, Total Annual Gallons = 425 gallons
Total Annual Gas Cost = \$1,488

Hybrid, Total Annual Gallons = 289 gallons
Total Annual Gas Cost = \$1,012

The salesperson drives $4,000 + 11,000 = 15,000$ miles each year. If they kept the car for 12 years, the car would only have 180,000 miles on it, which is less than 200,000. This means they'd be able to keep the car for the entire 12 years.

$$\begin{aligned}\text{Total Cost after 12 Years, Standard:} \\ 26,000 + 1,488(12) &= \$43,856\end{aligned}$$

$$\begin{aligned}\text{Total Cost after 12 Years, Hybrid:} \\ 28,000 + 1,012(12) &= \$40,144\end{aligned}$$

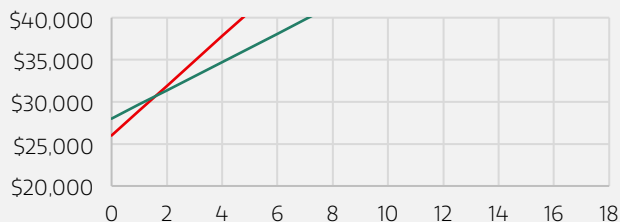
If the salesperson owned the car for all 12 years, they'd save more than \$3,000 by choosing the hybrid engine.

UBER/LYFT DRIVER

Standard, Total Annual Gallons = 842 gallons
Total Annual Gas Cost = \$2,947

Hybrid, Total Annual Gallons = 481 gallons
Total Annual Gas Cost = \$1,684

$$\begin{aligned}\text{Standard Cost} &= \text{Hybrid Cost} \\ 26,000 + 2,947 &= 28,000 + 1,684x \\ 1,263x &= 2,000 \\ x &= 1.6 \text{ years}\end{aligned}$$



It'll take 1.3 years for the hybrid to become the better deal. Since the average car lasts 12 years, the Lyft/Uber driver would save a lot of money by choosing the hybrid engine.

Standard, Total Annual Gallons = 842 gallons
Total Annual Gas Cost = \$2,947

Hybrid, Total Annual Gallons = 481 gallons
Total Annual Gas Cost = \$1,684

The Lyft/Uber driver drives $20,000 + 5,000 = 25,000$ miles each year. If they kept the car for 12 years, the car would have 300,000 miles on it. However, the car is only expected to last for 200,000 miles, so they won't be able to keep the car for 12 years. Instead, if they drive 25,000 miles each year, then they'll only keep it for 8 years.

$$\begin{aligned}\text{Total Cost after 8 Years, Standard:} \\ 26,000 + 2,947(8) &= \$49,576\end{aligned}$$

$$\begin{aligned}\text{Total Cost after 8 Years, Hybrid:} \\ 28,000 + 1,684(8) &= \$41,472\end{aligned}$$

If the Uber/Lyft driver used the car for all 8 years, they'd save more than \$8,000 by choosing the hybrid engine.

- 4 When people are shopping for a new car, their decision is often influenced by how much gas costs at the time. Watch the videos about the types of cars that were popular in years when the price of gas was relatively high versus when the price was relatively low.

- a How does the price of gas impact what kind of vehicle people decide to buy, and why do you think this is?

According to the video, hybrid cars were popular when gas was expensive (such as in 2012). When gas gets cheaper (like in 2015), people opt for vehicles like trucks and SUVs.

It seems that car shoppers are sensitive to the price of gas. When gas is expensive, they prefer cars that get better fuel economy such as hybrids, which allows them to save money at the pump. But when gas is inexpensive, drivers prefer larger and less fuel-efficient vehicles.

- b Even when the price of gas is low, can you think of any reasons why someone should still buy a hybrid?

Students' responses will vary. Sample response:

Even if the price of gasoline is low *now*, it doesn't mean that it'll stay low. If there's a chance that the price of gasoline will go up after someone buys a car, it may be a smart idea to choose a hybrid engine.

At the same time, the decision about which kind of car to buy doesn't only have to be financial. A hybrid engine burns less gas than does a standard engine, which means it's better for the environment. If someone cares a lot about this (and can afford the extra gas), they might prefer to purchase a hybrid even when the price of gas is low.