

### REAL-WORLD TAKEAWAYS

- On average, less than 100% of passengers show up for flights. Airlines often overbook flights so that they're not missing out on revenue on empty seats.
- If more passengers show up for the flight than there are seats on the plane, then some passengers may be involuntarily "bumped." If this happens, the airline is required by law to pay them (in addition to rebooking them on another flight).
- Even with the penalties, airlines typically expect to make more money by overbooking the flights than by not.

### MATHEMATICAL OBJECTIVES

- Calculate expected value and use it in decision making
- Calculate combinations and compound probabilities in a real-world context

### LESSON AT A GLANCE

120 MIN.

LESSON AT A GLANCE			120 MIN.
LAUNCH		Watch a video about Southwest Airlines in which more passengers show up for the flight than there are seats on the plane.	10 min.
PART ONE	1	Calculate the compound probability that all passengers show up for a flight, given a 10% no-show rate.	15 min.
	2	Write and solve an equation to determine how many tickets would need to be sold for the airline to expect a full flight.	15 min.
	3	Calculate the (tiny) probability of the flight being exactly full when the airline sells enough tickets to expect a full flight; discuss whether it's "better" for the airline to sell only as many seats as are on the plane or to overbook.	30 min.
PART TWO	4	Calculate the expected value of the airline's payout to travelers who are bumped from their flight when it's overbooked.	35 min.
	5	Debate whether airlines should be allowed to overbook flights.	15 min.

- L Students watch a clip from a TV show about the daily life of Southwest Airlines. In it, a gate agent is trying to find a volunteer to give up their seat for an overbooked flight, i.e. a flight for which the airline sold more tickets than there were seats.

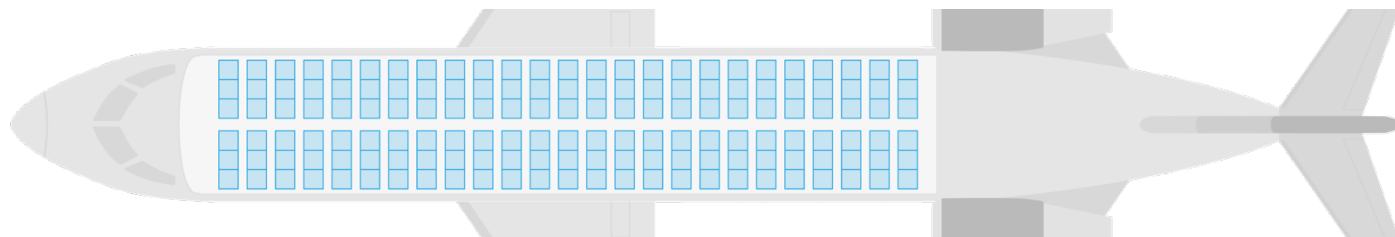
#### DISCUSSION QUESTIONS

1. What's happening in the video?
2. What does it mean for an airline to overbook a flight? Did you know they were allowed to do this?
3. If nobody volunteers to give up their seat, what do you think will happen?
4. Why do you think airlines overbook their flights? Can you think of any other industries that do this?

#### KEY TAKEAWAYS

- It's common for airlines to overbook their flights. This means selling more tickets than there are seats on the plane.
- If more passengers show up for the flight than there are seats, the airline will ask for volunteers to take a later flight (and will offer some kind of incentive). If nobody volunteers, the airline can force people to give up their seats.
- Industries that overbook include airlines, hotels, and rental cars.

- 1 For various reasons – e.g. a change in plans, missed connection – passengers don’t always show up for their flights. According to some estimates, the average no-show rate is around 10%. If an airline sold 150 tickets for a 150-seat plane, what’s the probability that the flight would be full?



If the no-show rate is 10%, then each person has a 90% chance of showing up. If each passenger has a 90% chance of showing up for the flight, then the probability that all 150 people show up is:

$$\underline{0.9} \cdot \underline{0.9} \cdot \underline{0.9} \cdot \underline{0.9} \dots 150 \text{ times} = (0.9)^{150} = 0.000000137$$

If the airline sells 150 tickets for a 150-seat plane, there’s a 0.000000137 (or a roughly 1 in 7 million) chance that everyone shows up for the flight. Practically, if every individual passenger has a 10% chance of missing the flight, it’s more or less guaranteed that the plane will *not* be full.

#### TEACHING TIP

Deep-thinking students might ask, “If 10% of all passengers are expected to miss the flight, does this necessarily mean that *each* passenger has a 10% chance? After all, don’t some passengers travel together?” If a student asks this, give them a round of applause! Since passengers often travel in groups – families, work colleagues, etc. – passengers aren’t necessarily independent from one another. For simplicity, though, the question assumes independence.

Otherwise, if students are familiar with compound probability, this question may be straightforward. However, Question 3 involves combinations and is substantially more difficult. To help foreshadow it, consider asking them whether “order matters” here. Students should agree that, in the case where 150 of 150 people make the flight, it does not matter in what order passengers arrive at the gate. Every permutation results in the same outcome, i.e. *150 choose 150 = 1*.

- 2 Let's assume the price of a one-way ticket is \$350. If the airline wanted the flight to be full, how many tickets would you recommend that it sell, and how much more money would it make by overbooking the flight?

**METHOD 1: REMOVE 10% OF PASSENGERS**

Let  $t$  = number of tickets sold

Need to find the value of  $t$  that,  
when we remove 10%, leaves 150 passengers.

$$t - (0.1 \cdot t) = 150$$

$$0.9 \cdot t = 150$$

$$t = 166.7$$

**METHOD 2: KEEP 90% OF PASSENGERS**

Let  $t$  = number of tickets sold

Need to find the value of  $t$  that,  
when we keep 90%, leaves 150 passengers.

$$0.9 \cdot t = 150$$

$$t = 166.7$$

Since it's not possible to sell 166.7 tickets, and since the airline wouldn't want more than 150 people to show up, the airline should sell 166 tickets (rather than 167 tickets). By selling 16 additional seats, the airline would generate an additional  $\$350 \cdot 16 = \$5,600$  in revenue.

- 3 Imagine the airline sells the number of tickets you suggested. Calculate the probability that *exactly* 150 passengers show up for the flight. What do you think should happen if more than 150 passengers show up?

$$\text{probability}(\text{make flight}) = 90\% = 0.9$$

$$\text{probability}(\text{miss flight}) = 10\% = 0.1$$

To determine the probability that exactly 150 of the 166 passengers show up, we can calculate  $P = (0.9)^{150} \cdot (0.1)^{16}$ . In this calculation, however, order doesn't matter. Since there are many ways for 150 passengers to make the flight and 16 passengers to miss the flight, we need to consider all the possible ways to arrange 150 makes and 16 misses.

#### METHOD 1: WAYS TO ARRANGE 150 "SHOWS"

ways to select 150 "shows" from 166 people

$$\frac{166 \cdot 165 \cdot 164 \cdot \dots 17}{150 \cdot 149 \cdot 148 \cdot \dots 1} = \frac{166 \cdot 165 \cdot 164 \cdot \dots 151}{16 \cdot 15 \cdot 14 \cdot \dots 1}$$

ways to arrange these 150 "shows"

#### METHOD 2: WAYS TO ARRANGE 166 TICKETHOLDERS

ways to arrange all 166 ticketholders

$$\frac{166 \cdot 165 \cdot 164 \cdot \dots 1}{(150 \cdot 149 \cdot 148 \cdot \dots 1) \cdot (16 \cdot 15 \cdot 14 \cdot \dots 1)} = \frac{166!}{(150!) \cdot (16!)}$$

possible ways to arrange 150 "shows"

possible ways to arrange 16 "no-shows"

P(exactly 150 people show up for flight):

$$0.9^{150} \cdot 0.1^{16} \cdot \frac{166!}{(150!) \cdot (16!)} = 0.1031$$

If the airline sells 166 tickets for a plane with 150 seats, there's a 10.31% chance that exactly 150 passengers show up.  
*Answers will vary about what students think should happen if more than 150 passengers show up for the flight.*

#### TEACHING TIP

Even if students have experience with permutations (order matters) and combinations (order doesn't matter), they might not immediately recognize that they need to calculate the combination *166 choose 150*. In this case, consider doing a quick mini-review using a smaller number of passengers. For instance, if a plane has 3 seats and the airline sells 5 tickets, how many different ways are there to select the three passengers who make the flight? The goal is for students to recognize that while A-B-C-D-E is the same outcome as B-C-A-E-D, it is a different outcome than C-D-E-B-A.

- 4 The Passenger Bill of Rights requires airlines to compensate passengers who are involuntarily bumped from flights. Depending on the length of the delay and the price of the ticket, compensation can be up to \$1,550. Watch the video about this.

Then, imagine an airline sells 166 tickets for a 150-seat flight. For each number of “show ups,” calculate the probability that exactly this many people arrive for the flight, and the amount the airline would expect to pay in compensation. Based on this, do you think it’s a smart financial decision for the airline to overbook the flight?

SHOW-UPS	151	152	153	154	155	156	157	158
COMPENSATION	\$1,550	\$3,100	\$4,650	\$6,200	\$7,750	\$9,300	\$10,850	\$12,400
PROBABILITY	0.0983	0.0873	0.0719	0.0546	0.0381	0.0242	0.0138	0.0071
EXPECTED COMP.	\$152	\$271	\$334	\$339	\$295	\$225	\$150	\$88
SHOW-UPS	159	160	161	162	163	164	165	166
COMPENSATION	\$13,950	\$15,500	\$17,050	\$18,600	\$20,150	\$21,700	\$23,250	\$24,800
PROBABILITY	0.0032	0.0013	0.0004	0.0001	basically 0	basically 0	basically 0	basically 0
EXPECTED COMP.	\$45	\$20	\$7	\$2	\$0	\$0	\$0	\$0

Sample Calculation: P(exactly 159 of the 166 people show up for the flight)

$$0.9^{159} \cdot 0.1^7 \cdot \frac{166!}{(159!) \cdot (7!)} = 0.0032$$

There is as 0.0032 (0.32%) chance that exactly 159 of the 166 ticketholders show up for the 150-seat flight. If this happens, 9 ticketholders would be bumped, and the airline would have to pay a penalty of  $\$1,550 \cdot 9 = \$13,950$ . Since the chance of this happening is only 0.0032, though, the compensation the airline expects to pay for this outcome is only  $0.0032 \cdot \$13,950 = \$45$ .

When the airline adds up the expected payouts for all of the possible outcomes (151 passengers showing up, 152 passengers showing up, etc.), it calculates a total expected payout of  $\$152 + \$271 + \$334 + \dots + \$0 = \$1,928$ . This is a lot of money that the airline expects to pay in compensation. However, by selling 16 extra tickets – or really, by selling 16 seats twice – the airline earns a guaranteed additional  $\$350 \cdot 16 = \$5,600$  in revenue. Based on this, it makes financial sense for the airline to oversell the flight.

- 5 Some people think it's unfair for airlines to sell more tickets than there are seats. Others argue it's a necessary part of the airline business. Do you think airlines should be allowed to overbook flights? Why or why not?

#### Possible Arguments in Favor of Overbooking

- Generates additional revenue for the airline; since airlines are publicly owned companies accountable to their shareholders, airline executives may not have much of a choice about whether they overbook or not (although Southwest Airlines doesn't overbook its flights, so airlines do have some discretion)
- By generating more revenue for the airline, overbooking *might* cause airlines to offer lower fares.
- If airlines didn't overbook flights, they'd need more airplanes to carry all of their passengers. This would be bad for the environment.

#### Possible Arguments Against Overbooking

- Overbooking is very disruptive to passengers who get bumped. Passengers who are bumped may be less likely to fly on that airline in the future.
- Overbooking can make the airline look bad, especially in the age of social media where people can easily share their frustrations with millions of others.
- Depending on how airlines decide which passengers to bump, it might unfairly prioritize some customers (e.g. first class) over others (e.g. those traveling in coach).