

REAL-WORLD TAKEAWAYS

- Major League Baseball stadiums are not standardized. The outfield walls have different heights and distances from home plate.
- If baseball stadiums were standardized, there would be upsides and downsides.

MATHEMATICAL OBJECTIVES

- Write and solve quadratic equations in a real-world context
- Evaluate a quadratic function at given values
- Sketch a quadratic function from an equation written in vertex form, and interpret the key features

LESSON AT A GLANCE

120 MIN.

LESSON AT A GLANCE			120 MIN.
LAUNCH		Watch a clip of a home run hit in Boston's Fenway Park, and discuss why Fenway's left field wall is so tall. Share prior knowledge about baseball and clarify what it means for a hit to be a "home run."	5 min.
PART ONE	1	Given quadratic equations modeling two homerun trajectories, sketch each and identify and interpret key features of the graph.	30 min.
	2	Evaluate each quadratic equation at a given distance to determine whether it would have been a homerun at various MLB ballparks.	20 min.
PART TWO	3	Write a quadratic equation modeling the trajectory of the average homerun.	40 min.
	4	Given the height and distance of the left field wall in every MLB ballpark, discuss where it would be easiest/hardest to hit a homerun.	15 min.
	5	Learn about the history of MLB stadiums. Debate whether MLB should standardize playing fields.	10 min.

- L** Students watch a video in which the former Boston Red Sox star David Ortiz hits a home run over the left field wall at Fenway Park. Baseball fans may recognize it as the “Green Monster,” a wall that’s much taller than those in other stadiums. (The reason is that when Fenway Park was built, there was a parking lot behind left field that prevented the stadium from being a normal size. To compensate for the shortened left side of the stadium and prevent pop-flies from being home runs, architects built the Green Monster to make Fenway “play like” a more normal-sized field.)

DISCUSSION QUESTIONS

1. Do you recognize the stadium in the video? What is the name of the stadium, and where is it?
2. Do you notice anything unique about the stadium’s wall?
3. Why do you think the designers of Fenway made the left field wall so tall?

KEY TAKEAWAYS

- Unlike with other sports, baseball fields are not standardized.
- The closer a wall is, the easier it will be for players to hit home runs. To prevent easy home runs, the designers of baseball stadiums can adjust both the distance and height of the outfield walls.

- 1 The left field wall in Boston's Fenway Park – aka the “Green Monster” – is 310 feet from home plate and 37 feet tall. Imagine a right-handed batter hits two balls down the left field line. Ignoring air resistance, the height of each ball can be modeled using the equations below, where x represents the ball's distance from home plate.

Identify the key points along each ball's flight path. Will either ball clear the Green Monster and result in a home run?

	TRAJECTORY, HIT A	TRAJECTORY, HIT B
	height = $-0.0003(x - 200)^2 + 15$	height = $-0.004(x - 170)^2 + 119$
HEIGHT AT CONTACT	<p>when ball is hit, know that distance = 0 feet</p> $y = -0.0003(x - 200)^2 + 15$ $y = -0.0003(0 - 200)^2 + 15$ $y = 3$ <p><u>height at contact = 3 feet</u></p>	<p>when ball is hit, know that distance = 0 feet</p> $y = -0.004(x - 170)^2 + 119$ $y = -0.004(0 - 170)^2 + 119$ $y = 3.4$ <p><u>height = 3.4 feet</u></p>
MAXIMUM HEIGHT	<p>In the form $y = -a(x - h)^2 + k$, the max value will always be k. This is because the $-a(x - h)^2$ term will always be negative, since it involves a negative coefficient ($-a$) being multiplied by a positive parenthetical term (because of the squaring).</p>	
	maximum height = 15 feet	maximum height = 119 feet.
DISTANCE @ MAX. HEIGHT	<p>want to know distance when height = 15 ft.</p> $y = -0.0003(x - 200)^2 + 15$ $15 = -0.0003(x - 200)^2 + 15$ $0 = -0.0003(x - 200)^2$ <p>when $x = 200$, $y = 0$</p> <p>distance at max. height = 200 feet</p>	<p>want to know distance when height = 119 feet</p> $y = -0.004(x - 170)^2 + 119$ $119 = -0.004(x - 170)^2 + 119$ $0 = -0.004(x - 170)^2$ <p>when $x = 170$, $y = 0$</p> <p>distance at max. height = 170 feet</p>

LANDING DISTANCE

want to know distance
when height = 0 feet

$$\begin{aligned}y &= -0.0003(x - 200)^2 + 15 \\0 &= -0.0003(x - 200)^2 + 15 \\0 &= -0.0003(x - 200)(x - 200) + 15 \\0 &= -0.0003(x^2 - 400x + 40,000) + 15 \\0 &= -0.0003x^2 + 0.12x + 12 + 15 \\0 &= -0.0003x^2 + 0.12x + 27\end{aligned}$$

solve using quadratic equation,
where $y = ax^2 + bx + c$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(0.12) \pm \sqrt{(0.12)^2 - 4(-0.0003)(27)}}{2(-0.0003)}$$

$$x = -23.6 \text{ and } 423.6$$

(-23.6 feet doesn't have
meaning in this context)

landing distance = 423.6 feet

want to know distance
when height = 0 feet

$$\begin{aligned}y &= -0.004(x - 170)^2 + 119 \\0 &= -0.004(x - 170)^2 + 119 \\0 &= -0.004(x - 170)(x - 170) + 119 \\0 &= -0.004(x^2 - 340x + 28,900) + 119 \\0 &= -0.004x^2 + 1.36x - 115.6 + 119 \\0 &= -0.004x^2 + 1.36x + 3.4\end{aligned}$$

solve using quadratic equation,
where $y = ax^2 + bx + c$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(1.36) \pm \sqrt{(1.36)^2 - 4(-0.004)(3.4)}}{2(-0.004)}$$

$$x = -2.5 \text{ and } 342.5$$

(-2.5 feet doesn't have
meaning in this context)

landing distance = 342.5 feet

HOME RUN?

want to know height
when distance = 310 feet

$$\begin{aligned}y &= -0.0003(x - 200)^2 + 15 \\y &= -0.0003(310 - 200)^2 + 15 \\y &= -0.0003(110)^2 + 15 \\y &= -0.0003(12,100) + 15 \\y &= -3.63 + 15 \\y &= 11.37\end{aligned}$$

When the ball is 310 feet from plate, its
height is 11.37 feet. The wall is 37 feet tall.

Not a home run.

want to know height
when distance = 310 feet

$$\begin{aligned}y &= -0.004(x - 170)^2 + 119 \\y &= -0.004(310 - 170)^2 + 119 \\y &= -0.004(140)^2 + 119 \\y &= -0.004(19,600) + 119 \\y &= -78.4 + 119 \\y &= 40.6\end{aligned}$$

When the ball is 310 feet from plate, its
height is 11.37 feet. The wall is 37 feet tall.

Home run!

2 In most pro sports, the dimensions of the playing surfaces are always the same. In the NFL, every field is 100 yards long and 160 feet wide. In the NHL, every rink is 200 feet long and 80 feet wide. In Major League Baseball, however, every field is different.

Using the earlier trajectories, determine whether the hits would clear the left-field walls of three other MLB stadiums. If you were a right-handed batter, which of the four stadiums – BOS, NYY, SF, or CHI – would you rather play in and why?

DISTANCES AND HEIGHTS, MLB LEFT-FIELD WALLS

NY YANKEES	SF GIANTS	CHI CUBS
318 feet, 8 feet	339 feet, 8 feet	355 feet, 16 feet

TRAJECTORY, HIT A

Want to know height
when distance = 318 feet

$$\begin{aligned}y &= -0.0003(x - 200)^2 + 15 \\y &= -0.0003(318 - 200)^2 + 15 \\y &= 10.8\end{aligned}$$

10.8 feet > 8 feet
Home run!

Want to know height
when distance = 339 feet

$$\begin{aligned}y &= -0.0003(x - 200)^2 + 15 \\y &= -0.0003(339 - 200)^2 + 15 \\y &= 9.2\end{aligned}$$

9.2 feet > 8 feet
Home run!

Want to know height
when distance = 355 feet

$$\begin{aligned}y &= -0.0003(x - 200)^2 + 15 \\y &= -0.0003(355 - 200)^2 + 15 \\y &= 7.8\end{aligned}$$

7.8 feet < 16 feet
Not a home run

TRAJECTORY, HIT B

Want to know height
when distance = 318 feet

$$\begin{aligned}y &= -0.004(x - 170)^2 + 119 \\y &= -0.004(318 - 170)^2 + 119 \\y &= 31.4\end{aligned}$$

31.4 feet > 8 feet
Home run!

Want to know height
when distance = 339 feet

$$\begin{aligned}y &= -0.004(x - 170)^2 + 119 \\y &= -0.004(339 - 170)^2 + 119 \\y &= 4.8\end{aligned}$$

4.8 feet < 8 feet
Not a home run

Want to know height
when distance = 355 feet

$$\begin{aligned}y &= -0.004(x - 170)^2 + 119 \\y &= -0.004(355 - 170)^2 + 119 \\y &= -17.9\end{aligned}$$

ball has already hit ground
Not a home run

Of the four stadiums (BOS, NYY, SF, and CHI), Yankee Stadium is the only one in which both trajectories would result in a home. At least based on the two sample trajectories, this appears to be the easiest stadium for home runs for right-handed batters.

TEACHING TIP

Students may ask why we’re focusing on the left field wall (instead of the centerfield wall and/or the right field wall). Most professional baseball players are right-handed. Thus, they’ll hit most of the balls towards left field.

- 3 According to the statistics website Statcast, the average home run (left field) travels a distance of 378 feet and reaches a maximum height of 81 feet. Assuming the ball is struck from 3 feet in the air, write an equation to model the average home run ball's trajectory.

Students' approaches will vary. Sample approach:

STEP 1: WRITE DOWN KNOWN INFORMATION

- know vertex form of parabola, $y = a(x - h)^2 + k$
- know max. height = 81 feet; $y = a(x - h)^2 + 81$
- know that when distance = 0 feet, height = 3 feet;
 $3 = a(0 - h)^2 + 81$
- know that when distance = 378 feet, height = 0 feet;
 $0 = a(378 - h)^2 + 81$
- Want to find values of a and h

STEP 2: WRITE A IN TERMS OF H (OR VICE-VERSA)

From step 1, have two equations and two unknowns. Can rewrite one in terms of other. Will rewrite a in terms of h . To do this, will use $3 = a(0 - h)^2 + 81$ equation b/c it's easier.

$$3 = a(0 - h)^2 + 81$$

$$3 = a(-h)^2 + 81$$

$$3 = -ah^2 + 81$$

$$-78 = -ah^2$$

$$a = \frac{-78}{h^2}$$

STEP 3: SUBSTITUTE FOR A, SOLVE FOR H

Since we used $3 = a(0 - h)^2 + 81$ to rewrite a in terms of h , we must use the *other* (a, h) equation to solve for h .

$$0 = a(378 - h)^2 + 81$$

$$0 = \left(\frac{-78}{h^2}\right)(378 - h)^2 + 81$$

multiply each term by h^2 to remove fraction

$$0 = -78(378 - h)^2 + 81h^2$$

$$0 = -78(378 - h)(378 - h) + 81h^2$$

$$0 = -78(142,884 - 756h + h^2) + 81h^2$$

$$0 = -11,144,952 + 58,968h - 78h^2 + 81h^2$$

$$0 = 3h^2 + 58,968h - 11,144,952$$

$$0 = h^2 + 19,656h - 3,714,984$$

$$h = \frac{-(19,656) \pm \sqrt{(19,656)^2 - 4(1)(-3,714,984)}}{2(1)}$$

$$h = -19,843.2 \text{ and } 187.2$$

In this context, only 187.2 feet makes sense

$$h = 187.2$$

STEP 4: SOLVE FOR A, WRITE EQUATION

from Step 2, know $a = \frac{-78}{h^2}$; from Step 3, know $h = 187.2$

$$a = \frac{-78}{(187.2)^2} = -0.00223$$

Now that we know the values for a , h , and k , we can write the equation for the trajectory of the average MLB home run:

$$\text{Height} = a(x - h)^2 + k$$

$$\text{height} = -0.00223(x - 187.2)^2 + 81$$

TEACHING TIP

This is a challenging question. Encourage students to write down everything they know – referring back to Question 1 should help – and everything they want to find out. Once students know what they're looking for (the values of a and h), encourage them to follow their noses and see if they come up with anything. With enough hard work and strategic nudges, students should be able to find the height equation.

That said, if students are unable to make progress, you can modify the problem to make it slightly easier. One of the hardest steps in the process is finding the value of h , i.e. the distance at which the ball reaches its maximum height. The reason this is challenging is because the ball is hit from the air, not the ground. As a result, unlike with a parabola that starts at the origin, the vertex of the baseball trajectory is *not* exactly halfway between where the ball is hit ($x = 0$ feet) and where the ball lands ($x = 378$ feet).

However, if we imagined that the ball *were* hit from the ground – more of a golf shot than a baseball swing – we could simplify the process of finding h considerably. In this case, the value of h would simply be the halfway point between 0 feet and 378 feet, or 189 feet. Once we have this, the trajectory equation becomes $0 = a(0 - 189)^2 + 81$. When we solve for a , we get a value of -0.00227 and a final equation of height = $-0.00227(x - 189)^2 + 81$. Even though this is different than the more accurate $-0.00223(x - 187.2)^2 + 81$, it's still really close.

- 4 The following table shows the distance and height of every left field wall in Major League Baseball. Using your home run trajectory equation, determine which stadium is the easiest for home runs and which stadium is the hardest. Do you think this is fair?

TEAM	DIST	HGHT	TEAM	DIST	HGHT	TEAM	DIST	HGHT
Angels (LAA)	330'	5'	Giants (SF)	339'	8'	Rangers (TEX)	332'	14'
Astros (HOU)	315'	21'	Guardians (CLE)	325'	19'	Rays (TB)	315'	11'
Athletics (OAK)	330'	8'	Mariners (SEA)	331'	15'	Red Sox (BOS)	310'	37'
Blue Jays (TOR)	328'	8'	Marlins (MIA)	344'	7'	Reds (CIN)	328'	12'
Braves (ATL)	335'	6'	Mets (NYM)	335'	8'	Rockies (COL)	347'	8'
Brewers (MIL)	344'	8'	Nationals (WAS)	336'	9'	Royals (KC)	330'	9'
Cardinals (STL)	336'	8'	Orioles (BAL)	333'	7'	Tigers (DET)	345'	7'
Cubs (CHC)	355'	16'	Padres (SD)	334'	4'	Twins (MIN)	339'	8'
Diamondbacks (ARI)	330'	8'	Phillies (PHI)	329'	11'	White Sox (CWS)	335'	8'
Dodgers (LAD)	330'	4'	Pirates (PIT)	325'	6'	Yankees (NYY)	318'	8'

For the average MLB left-field home run, the following table shows the height of the ball when it reaches the left-field wall of each team's stadium (the first number) as well as how much higher the ball is than the wall (the second number).

Angels (LAA)	35.5'	↑ 30.5'	Giants (SF)	29.6	↑ 21.6'	Rangers (TEX)	34.2'	↑ 30.2'
Astros (HOU)	44.6'	↑ 23.6'	Guardians (CLE)	38.7'	↑ 19.7'	Rays (TB)	44.6'	↑ 33.6'
Athletics (OAK)	35.5'	↑ 27.5'	Mariners (SEA)	34.9'	↑ 19.9'	Red Sox (BOS)	47.4'	↑ 10.4'
Blue Jays (TOR)	36.8'	↑ 28.8'	Marlins (MIA)	26.2'	↑ 19.2'	Reds (CIN)	36.8'	↑ 24.8'
Braves (ATL)	32.3'	↑ 26.3'	Mets (NYM)	32.3'	↑ 24.3'	Rockies (COL)	24.1'	↑ 16.1'
Brewers (MIL)	26.2'	↑ 18.2'	Nationals (WAS)	31.6'	↑ 22.6'	Royals (KC)	35.5'	↑ 26.5'
Cardinals (STL)	31.6'	↑ 23.6'	Orioles (BAL)	33.6'	↑ 26.6'	Tigers (DET)	25.5'	↑ 18.5'
Cubs (CHC)	18.2'	↑ 2.2'	Padres (SD)	32.9'	↑ 28.9'	Twins (MIN)	29.6'	↑ 21.6'
Diamondbacks (ARI)	35.5'	↑ 27.5'	Phillies (PHI)	36.2'	↑ 25.2'	White Sox (CWS)	32.3'	↑ 24.3'
Dodgers (LAD)	35.5'	↑ 31.5'	Pirates (PIT)	38.7'	↑ 32.7'	Yankees (NYY)	42.8'	↑ 34.8'

The easiest stadium for home runs is Yankee Stadium. There, the average home run would clear the left field wall by 34.8 feet. The hardest stadium for home runs is Chicago's Wrigley Field. There, the ball would only clear the left field wall by 2.2 feet.

Students' answers will vary about whether this difference is fair.

- 5 Many people wonder why baseball fields vary so much by stadium. Watch the video about the history of MLB stadiums. Do you think the dimensions of baseball fields should be standardized, or do you think it's okay for every stadium to be unique? Explain.

Students' responses will vary. Possible arguments include:

Arguments in Favor of Standardizing Dimensions

- makes it easier to compare players from different teams, e.g. when determining who's the best home run hitter or which pitcher is the best against the long ball; this could make salary negotiations fairer
- since each team plays half of its regular-season games in an away stadium (81 of 162), standardized dimensions could remove any advantage teams with difficult stadiums currently have; this could make the process fairer for determining which teams advance to the post-season

Arguments Against Standardizing Dimensions

- because each baseball stadium is unique, it's fun for fans to visit different stadiums
- unlike other sports (football, soccer, hockey, tennis), the playing surface of baseball is different in every stadium; this gives baseball a unique charm that other sports lack
- the non-standard dimensions are rooted in the city's history; stadiums like Fenway are shaped the way they are to accommodate the surrounding neighborhood, which creates an additional connection between team and city