



© Citizen Math 2020

REAL-WORLD TAKEAWAYS

- Humans beings' alertness changes throughout the day because of their natural circadian rhythm.
- A typical teenager's circadian rhythm is shifted from an adult's. Teenagers experience peak alertness later in the day.
- The typical work day matches adults' peak alertness; the typical high school day does not match teenagers' alertness.

MATHEMATICAL OBJECTIVES

- Identify key features of a graphed periodic function including: period, amplitude, and relative maximums and minimums
- · Interpret a trigonometric function modeling a real-world phenomenon and use it to justify decision-making

		LESSON AT A GLANCE	60 MIN.
LAUNCH		Students watch a short clip about the extreme measures one mother goes to in order to wake up her teenage daughter. Then they share their own experiences struggling to wake up early for school.	5 min.
PART ONE	1	Interpret the graph of a periodic function modeling the alertness of a typical adult over a 48-hour period.	15 min.
	2	Identify where on the alertness cycle a typical adult's workday falls and discuss the reasonableness of this schedule from the perspectives of both workers and employers.	10 min.
PART TWO	3	Compare the alertness cycle of the average teenager to that of the average adult. Discuss the implications for students' alertness during school hours.	15 min.
	4	Watch a video describing the pros and cons of starting middle and high school later in the morning. Debate the best time for school.	15 min.

L Students watch a video in which a mother describes her challenge waking up her 13-year-old daughter for school. Fortunately for the mom, a local radio station has a morning solution: They've hired a bugler from the local horserace track! As soon as he starts playing his horn, the teenager wakes up...then covers her head with her pillow and goes back to sleep.

DISCUSSION QUESTIONS

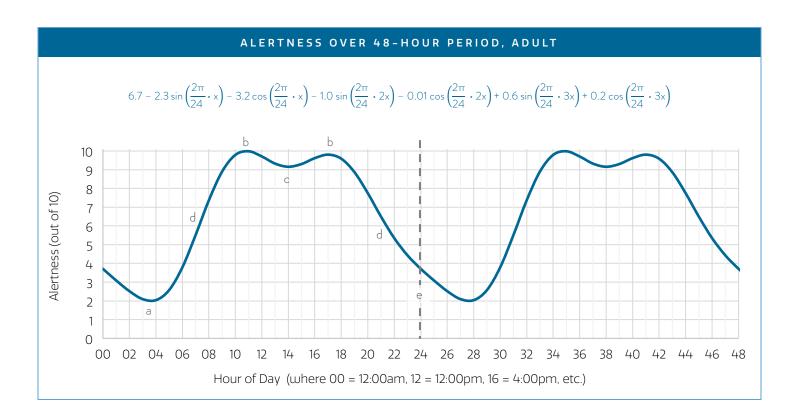
- 1. Do you have a hard time waking up for school? If so, what are some strategies you've tried to get out of bed?
- 2. If you could sleep as late as you wanted, what time do you think you'd wake up?
- 3. Did you have a hard time waking up when you were younger, or is this a pretty new phenomenon?
- 4. Are you sleepy all day, or are there parts of the day when you feel alert?

KEY TAKEAWAYS

- It can be difficult to wake up in the morning, especially if school starts early.
- While adults and young children often struggle to wake up on time, it can be especially hard for teenagers.

1 Most humans feel sleepy during some parts of the day and wide-awake during others. This is due to the body's circadian rhythm, the 24-hour cycle that regulates how alert we feel. For a typical adult, alertness can be roughly modeled by the function below.

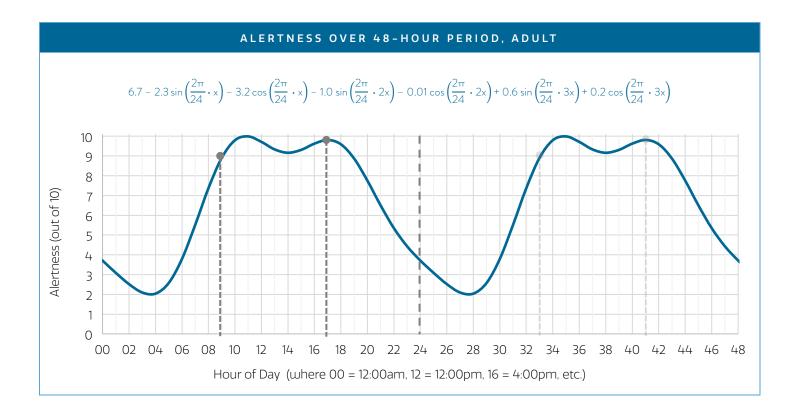
What are the key features that you notice in the graph?



Students' responses will vary. Observations may include:

- a. Alertness is at its lowest just before hour 04, or 4:00am. The typical adult is probably asleep at this time. Even when they're asleep, though, alertness isn't zero. This may indicate an evolutionary need to always be somewhat alert in case of danger.
- b. Alertness is highest around hours 11 (11:00am) and 17 (5:00pm).
- c. In the afternoon, alertness exhibits at dip at hour 14 (2:00pm). This could indicate that the typical adult doesn't eat lunch until 2pm. It could also indicate that they eat lunch earlier, in which case the dip likely corresponds to post-lunch drowsiness.
- d. Around hour 07 (7:00am), the alertness graph at its steepest positively. This indicates that the alertness is *changing the fastest* at this time, i.e. the adult is waking up the fastest. Around hour 21 (9:00pm), the graph is at its steepest negatively; at this time, the person is fading the fastest.
- e. Beginning at hour 24 i.e. midnight of the second day the cycle repeats itself.

2 In many countries, the typical workday begins at 9:00am and ends at 5:00pm. Considering the needs of both workers and employers, does this seem like a good schedule when it comes to alertness? Explain.



Students' responses will vary. Sample response:

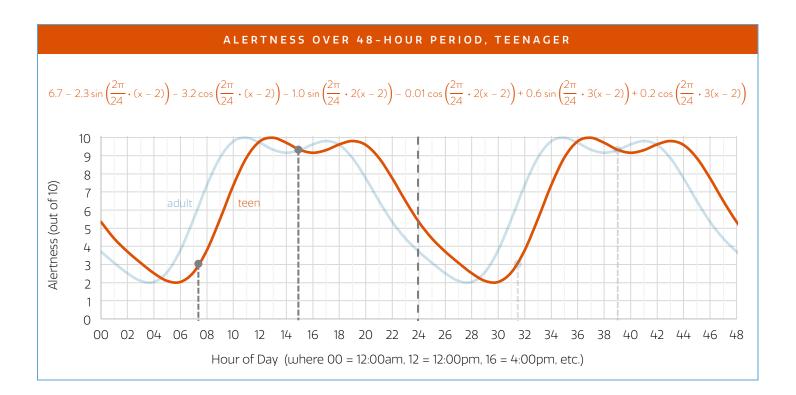
The typical workday begins at 9:00am. At this time, the typical adult isn't fully alert; their alertness doesn't reach its maximum level until closer to 11:00am. From the perspective of the employer, instead of starting work at 9:00am (and ending at 5:00pm), it would be more productive to start work at 11:00am (and end at 7:00pm).

However, workers might not like this change. Workers with kids might need to hire babysitters until they get home, which would be expensive. Not only that, but if adults didn't get home until later in the evening, they'd have less time to spend with their families, go out to dinner, or do many of the other things that are currently possible on a 9-to-5 schedule.

Based on the circadian rhythm of the typical adult, the typical workday isn't perfectly optimal from an alertness perspective (especially from the perspective of employers). But it's still pretty good, and it allows workers to have a reasonably long evening.

3 Teenagers have a different circadian rhythm than adults do; for the typical teen, alertness is roughly modeled by the function below. Teenagers also have a different schedule. While the typical workday runs from 9:00am to 5:00pm, for middle and high schoolers, the typical school day runs from 7:30am to 3:00pm.

Based on the graph below, how is the alertness cycle different for teens and adults, and would you say that school starts at the optimal time? If so, why? If not, what time do you think would be better?



Students' responses will vary. Sample response:

The shape of the alertness cycle is identical for teenagers and adults. However, the teen graph is shifted to the right by two hours. This means that teens start to become alert two hours later than adults do in the morning and get sleepy two hours later in the evening.

According to the graph, when school starts at 7:30am (hour 7.5), the typical teenager's alertness is approximately 3 out of 10; at this hour, they're still very sleepy. A teenager becomes more and more alert over the course of the morning, and their alertness reaches a maximum around hour 13 (1:00pm). At this point, though, the school day is almost over.

Based on this, 7:30am seems like a bad start-time when it comes to a teenager's circadian rhythm; such an early start effectively guarantees that students will still be sleepy when school starts. If the goal is to maximize teens' alertness during school hours, a better 7.5-hour schedule might run from hours 10-17:30 (10:00am-5:30pm) or even 12-19:30 (12:00-7:30pm). The downside of this, though, might mean less time for after-school sports, homework, or other activities.

4 Many sleep experts believe that middle school and high school should start no earlier than 8:30am. However, many others argue that changing the start time of school would be too disruptive to parents, businesses, and others. Watch the video about this debate.

Then describe some of the pros and cons of starting school later in the morning. If you were a decision-maker for your school/district, what start time would you recommend, and how would you balance the needs of both adults and students?

Upsides of Starting School Later

- Students will be more alert during the school day, which could result in more learning and better grades
- More time to eat breakfast in the morning
- Fewer suspensions for tardiness
- Fewer car accidents from drowsy driving
- More morning planning/prep time for teachers

Downsides of Starting School Later

- Challenge for working parents who drop students off at school before going to work
- Less time for after-school activites like sports, drama, etc.
- Later end-time could make it harder for local employers who rely on students for work
- Expensive for school districts who would need to purchase more buses. (Currently, elementary schools typically start later than middle and high schools, which allows districts to use the same buses for an early shift and a late shift.)

Responses will vary about what time school should start and how to balance the needs of adults and teens. Sample response:

If I were a decision-maker for my school or district, I would want the start time for middle and high schools to be later than 7:30am. When it comes to teenagers' alertness cycles, a closer-to-optimal school day would run from 12pm until 7:30pm. Not only would this maximize students' alertness, but it would still allow the district to use the same buses for elementary students and secondary students. However, such a late end-time might make it impossible for students to participate in sports, have after-school jobs, or participate in other important activities. A good compromise, then, might be for middle and high schools to start at 9:00, even if it required the school district to purchase more buses.

If parents, local businesses, and others opposed this plan, though, and if the only option was to keep the current 7:30am start time, I would want students' class schedules to account for their circadian rhythms. For instance, I would want any classes that could impact college admissions or that are tested at the end of the year (such as math and ELA) to happen later in the morning or in the early-afternoon. I would also want tests like the SAT, ACT, and AP to be given later in the day; after all, it wouldn't be fair to require a student to take the SAT when they're still sleepy. Finally, I might support policies that limited how much homework teachers could assign. According to the video, lots of teenagers don't get a full night of sleep. Changing the school's/district's homework policy might make it easier for teenagers to get a full night of sleep each evening.